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CONTRIBUTION OF ĀRYABHAṬĪYA IN THE FIELD OF MATHEMATICS AND ASTRONOMY: MODERN PERSPECTIVE[1]

Āryabhaṭa was the titan of the age and wonder of the Mathematical and Astronomical stage. Āryabhaṭa I (476-540 AD) wrote Āryabhaṭīyam in 499 AD at the age of 23 years[2]. He was born in 476 AD probable in Aśmaka[3] but according to Āryabhaṭīya he lived and acquired knowledge mostly in Kusumapura[4] near Patliputra (modern Patna). Āryabhaṭīyam text consists of four parts (Pādas)—1. Gītikāpāda 2. Gaṇitapāda 3. Kālakriyāpāda 4. Golapāda. The text mostly highlights the problems of Arithmetic, Geometry, Algebra, Trigonometry and Astronomy. The text deals in Alphabetic Numerical System, Simple and Quadratic equations, first degree indeterminate equations (kuṭṭaka), table of Trigonometrical calculations {Sines of allied angles as Sine (jyā), Cosine (koṭijyā)}, Natural numbers, Rules of Squares and Cubes and value of Pi (π). The approximate value of Pi (π) = 3.1416 which is universally accepted even today. In Āryabhaṭīya, Āryabhaṭa the great philosopher and first scientist of India, realized that "the earth is spherical[5] (circular in all directions). He used the 'Yuga theory' to expound the velocity of planets. We also find in Āryabhaṭīya

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[2] ṣaṣṭyabdānāṃ ṣaṣṭiryadā vyatītāstryaṣca yugapādaḥ ।
tryadhikā viṃśatirabdāstadeha mama janmano'tītāḥ ॥ 3.10 Kālakriyāpāda

[3] Indian Mathematics and Astronomy, p. 34

[4] brahmakuśaśibudhabhṛguvikujagurukoṇabhagaṇānnamaskṛtya ।
āryabhaṭastviha nigadati kusumapure'bhyarcitaṃ jñānam ॥ Gaṇitapāda, 1

[5] bhūgolaḥ sarvatovṛttaḥ, Golapāda, 6

a beautiful and scientific calculation for nomenclature (naming) of Week-days as Bhānuvāra, Somavāra etc.

The research paper is aimed at determining the Mathematical and Astronomical facts with examples and proofs from the Āryabhaṭīya and its commentaries. It is the need of time to co-relate the blend of ancient Indian Mathematics and Astronomy with that of universal modern Mathematics and Astronomy.

The importance of mathematics has been highlighted in the **Vedāṅga jyotiṣa** (1400 B.C.) of Lagadha:

> *Yathā Śikhā mayūrāṇāṁ nāgānāṁ maṇayo yathā* ।
> *Tadvat vedāṅga śāstrāṇāṁ gaṇitaṁ mūrdhani sthitam*[6] ॥

i.e. Like the crests on the heads of peacocks, like the gems on the heads of the cobras, Mathematics is at the top of the vedāṅga śāstrās.

Many commentators have contemplated and considered Āryabhaṭīya in their ways. Āryabhaṭīya was translated into Arabic as 'Zij-Al-Arjabhar' by Abdul Hassan Al-Ahwazi (8th Century AD). It was translated into latin in 13th Century AD. Rode (1879) translated Gaṇitapāda of Āryabhaṭīya into French in 1975. Kurṭa Alferic translated Gaṇitapāda of Āryabhaṭīya into German. First commentator of Āryabhaṭīya was Bhaskara I (600-680 AD), who was the pupil of Āryabhaṭa I. He contributed mostly in Algebra. Lall (720-790 AD) was disagreed with much Astronomical works. He accepted the value of Pi (π). He wrote commentary on Brahmgupta's 'Khanḍakhādyaka. Govindswamin (800-860 AD) wrote commentary on Mahābhāskarīya, an astronomical work of Bhaskara I. He considered Āryabhaṭīya's 'sine' tables and constructed a table which marked improved values. Shankara Narayan (840-900 AD) focused on Āryabhaṭīya through his commentary on Laghubhāskarīya of Bhāskara I. Among other commentators: Suryadeva Yajvan (1191 AD) gives alphabet numerals in his commentary, Parmeshwara (15th century AD), Yalla (1480), Nilakhantha somyaji (1444 AD) wrote

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[6] Vedāṅgajyotiṣa; 4

commentaries on Āryabhaṭīya which are Siddhāntadarpaṇa (that presents geometrical picture of planetary) and Tantra sangrah (that presents algebra geometry), Raghunath Raja (1597AD), Ghaṭigopa(1800 AD) etc. Modern researcher Roger Billard (1971) highlighted **'Yuga Theory'** in his paper entitled "Indian Astronomy: An investigator of Sanskrit texts and their numerical data".

Contribution of Āryabhaṭīya in alphabetical representation of numerals and numbers:

Āryabhaṭa invented ingenious method to represent alphabetic notation in Gītikāpāda-

varge'varge'vargākṣarāṇi kāt ṅmau yaḥ ।
khadvinavake svarā nava varge'varge navāntyavarge vā [7]॥

The varga letters (from ka to ma are to be written in the place value is (10 raised to the power which is an even number), a square number, the avarga letters (from ya to ha), 10 raised to the power which is an odd number, a non-square number. The varga letters from ka to ma take numerical values from 1 to 25. The numerical value of initial avarga letter ya is 30. Nine vowels are to be written from right to left so that each vowel represents two place values of (powers of 10 raised to even and odd numbers) square and non-square numbers respectively from right to left. This is shown in following table:

Varga Letters and represented numbers

Ka-varga	k-1	kha 2	g 3	gh 4	ṅ 5
Ca varga	c 6	ch 7	j 8	jh 9	ñ 10
ṭa varga	ṭa 11	ṭh 12	ḍ 13	ḍh 14	ṇ 15
Ta varga	t 16	th 17	d 18	dh 19	n 20
Pa varga	p 21	ph 22	b 23	bh 24	m 25

[7] Gītikāpāda; 2

Avarga:

Y	R	L	V	Ś	ṣ	S	H
30	40	50	60	70	80	90	100

Nine vowels (svara):

A	I	U	ṛ	Ḻ	E	ai	O	Au

In this alphabetic notation, vowels are equal whether short (hrasva) or long (dirgha). As for example , ka = kā = 1, ki = kī =100, ku = kū = 10000 and so on. The numbers can be represented upto 10^{18}.

The velocity of planets in a Yuga (1 Yuga = 43,20,000):

It is mentioned in the ślokās 3 & 4 of Gītikāpāda[8] of Āryabhaṭīya (the velocity is the the no. of revolutions).
As example:

1. Ravi (the Sun)------- khyaghṛ

= kh (u) + y (u) + gh (ṛ)
= 2 (10^4) + 30 (10^4) + 4 (10^6)
= 20000 + 300000+ 4000000
= 43, 20,000

2. Soma (the Moon)------- cayagiyiṅuśuchṛḷ

= c (a) + y (a) + g (i) + y (i) + ṅ (u) + ś (u) + ch (ṛ) + l (ḷ)
= 6 (1) + 30 (1) + 3 (10^2) + 30 (10^2) + 5 (10^4) + 70 (10^4)) + 7 (10^6)) + 50 (10^6)
= 6 + 30 + 300 + 3000 + 50000 + 700000 + 7000000 +50000000
= 5, 77, 53, 336

[8] Ibid. 3, 4

3. Bhūmi (the Earth)------- niśibuṇḷkhṣṛ
= ṅ (i) + ś (i) + b (u) + ṇ (ḷ) + (kh +ṣ) ṛ
= 5 (10^2) + 70 (10^2) + 23 (10^4) + 15 (10^8) + (2+80) X 10^6
= 500 + 7000 + 230000 + 1500000000 + 82000000
= 1,58,22,37, 500

There is clear reference of numbers in Āryabhaṭīya. In the Āryabhaṭīya we find eka (1), 10^1 to 10^9 {eka (1), daśa (10), śata (100), sahasra (1000), ayuta (10000), niyuta (100000), prayuta (1000000), koṭi (10000000), arbuda (100000000) and vṛnda (1000000000) } in the following śloka.

Ekaṁ daśa ca śatañca sahasramayutaniyute tathā prayutaṁ ।
koṭyarbudañca vṛindaṁ sthānātsthānaṁ daśaguṇaṁ syāt[9] ॥

Contribution in the Field of Trigonometry:

Trigonometry was an important gift of ancient mathematicians to the mathematical world. In modern time 'trikoṇamiti' Sanskrit world used for trigonometry which literary means "measurement of triangle". Āryabhaṭa I used *'jyā'* (sine), *'Koṭijyā' (cosine), 'utkramajyā' (versine) and 'autkramajyā (inverse sine).*

Āryabhaṭa provided the tables of sine, cosine and versine values at intervals of $90^0/24$ = 3.45 degrees. He clearly used the following trigonometric formula incidentally the same given by Newton.

[9] Gaṇitapāda; 2

Sin (n+1) x-sin nx=sin nx-sin(n-1) x-(1/225) sin nx

Āryabhaṭa sine differences	**Āryabhaṭa versine r- *Jyā* (90^0 - θ)**	**(Sin θ) X3438**	***Jyā* (Āryabhaṭa sine)**	**Arcs**	**S.N.**
	7	224.85	225	3.45	1
224	29	448.95	449	7.3	2
222	66	670.72	671	11.15	3
219	117	889.82	890	15	4
215	182	1105.01	1105	18.45	5
210	261	1315.01	1315	22.3	6
205	354	1520.58	1520	26.15	7
199	460	1719	1719	30	8
191	579	1910.05	1910	33.45	9
183	710	2092.09	2093	37.3	10
174	853	2266.08	2267	41.15	11
164	1007	2431.01	2431	45	12

This table of sines was used by Indian to calculate the relative distances between the Earth, Moon and Sun.

Āryabhaṭa I gave the lines of angle between Zero to Ninety (0-90). This was used by astrologers to decide the actual place of planets. Āryabhaṭa gives the method of calculating the dimension of a shadow cast by an object placed in the cone of the light coming out of a lamp or a source of illumination and by applying the rule of three in the geometry of triangles, he gives a simple rule in respect to these shadows. This forms the basis of calculating eclipses[10].

The value of pi (π)

Āryabhaṭa was the first to mention the most accurate value of Pi (π) which is correct to four decimal places. The ratio of the circumference of a circle to its diameter is a constant, denoted

[10] Āryabhaṭīyam; Gaṇitapāda, 14-16

by π. Its value is given by Āryabhaṭa I in the following śloka-

> *Caturadhikaṁ śatamaṣṭaguṇaṁ dvāṣaṣṭistathā sahasrāṇām* ।
> *Ayutdvayaviṣkambhasyāsanno vṛttapariṇāḥ*[11] ॥

i.e., If we add four (4) to one hundred (100), multiply it by eight (8) and add to sixty two thousand (62000) to that number, the result is approximately the circumference of a circle whose diameter is twenty thousand.

$$\textbf{Pi}\ (\pi) = \frac{\textit{Circumference}}{\textit{Diameter}} = \frac{62{,}832}{20{,}000} = 3.1416.$$

This value of *Pi* **(π)** has been universally accepted and widely applauded by the whole mathematicians even today.

Algebra:

Āryabhaṭa I has given two important methods to solve equations.

Indeterminate equations of the first degree: Kuṭṭaka--

We find the trace of indeterminate equations from the time of śulvasūtras. Greek mathematician Diophantus (3rd century) is given credit for solving indeterminate equations. The problem of finding solution in integers for X and Y in an equation of the from: ax + c = by where a, b and c are integers and it was given great importance by ancient mathematicians and astronomers. Āryabhaṭa was the first mathematician who solved indeterminate equations in integers in a systematic method. He also used it to solve the problems of determining the periods of the Sun, the Moon and the planets in astronomy[12]. The method of general solution of indeterminate equations of first degree in

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[11] Ibid. Gaṇitapāda, 10

[12] Gaṇitapāda, 10

positive integers developed by Āryabhaṭa is called **Kuṭṭaka** which literally means breaking or pulverizing. Bhāskar I who has explained the method elaborately with examples in his commentary on the Āryabhaṭīya.

Example:

Find the number which gives 5 as the remainder divided by 8, 4 as the remainder when divided by 9 and 1 as the remainder when divided by 7.
The problem is expressed algebraically in the following equation:

$$N = 8x+5 = 9y+4 = 7z+1$$

By the method of **Kuṭṭaka,** we get the least value of unknown number N is 85.

Quadratic equations:

Āryabhaṭa formulated the method for calculation of compound interest which provided the solution of quadratic equations firstly. Later, Shridharacharya (750 AD) elaborated the method for solving quadratic equation ($ax^2 + 2bx = c$). Āryabhaṭa says that the problem is “a principal amount (A) is lent for unknown monthly interest (x) and the unknown interest is lent out for interest for some period equal to (B). what is the rate of interest (x) on the principal amount (A).”

Āryabhaṭa gave the formula “multiply the sum of the interest on the principal and the interest on this interest by the time and by the principal. Add to this result the square of half the principal. Take the square root of this. Subtract half the principal and divided the remainder by the time. The result will be the interest on the principal.” This formula involves the solution of a quadratic equation in the form of ($ax^2 + 2bx = c$). the solution in modern notation.

$$x = \frac{\sqrt{B \text{ x } A \text{ x } T + \left(\frac{A}{2}\right) 2} - \frac{A}{2}}{T}$$

For example, the sum of 100 (A) is lent for one month. Then the interest received is lent for six months (T). At that time, the original interest plus the interest on this interest amounts to 16 (B).

$$\frac{\sqrt{16 \text{ x } 100 \text{ x } 6 + 2500} - 50}{\mathrm{T}} = 10$$

The interest received on principal 100 in one month 10[13].

Rule of three (trairāśika):

The trace of trairāśikā is found in yajurveda, vedanga jyotish etc. Āryabhaṭa provided the method of the 'trairāśikā' that is "phala X icchā / Pramāna" for finding x number with given three numbers. He also elaborated this rule to the rule of five, rule of seven etc. This rule was spread to Arab then to Europe.

Square roots and cube roots:

Śulvasūtras are the source of Square roots and cube roots. Āryabhaṭa described the methods for extraction of Square roots and cube roots. Which are purely based on decimal place value with zero.

Contribution of Āryabhaṭīya in the field of Astronomy:

Āryabhaṭa's view that the Earth and all the planets are rotating on their axis and following an elliptical orbit around the Sun. He explained a heliocentric solar system and considered the motions of all planets around the Sun. he maintained that "the Earth is spherical (circular in all directions)". **Bhūgolaḥ sarvatovṛttaḥ[14].** He gave a systematic treatment of the position of the planets in space. He also described that the orbits of the all planets around the Sun are ellipses. The heliocentric theory

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[13] Indian contribution to Mathematics and Astronomy, pp. 114-115

[14] Golapāda, 6

of gravitation much before Polish astronomer Copernicus (16^{th} century) and Galileo (16^{th} century).

Āryabhaṭa, first scientist of India who realized that the Earth is round and due to movement of the Earth (diurnal course) day and night occur continuously. The Moon and all planets are not self illuminating but scintillate by the reflected sunlight. He clearly explained the cause of eclipses of the Sun and the Moon. He showed that eclipses are caused by Earth shadow over The Moon and the Moon obscuring the Sun. He described numerical and geometrical rules for eclipse calculation. He calculated the size and extent of Earth's shadow and then provided the computation for the size of the eclipsed part. Āryabhaṭa calculated the diameter of Earth as 1050 yojanas. The circumference of the Earth can be shown as 1050 X π (3.1416) =3298.68. A yojana was equal to 8000 human heights or 13 kms. One human height = 6 feet. i.e. 8000 human height or 13Km = 48000 feet and according to this calculation 1 km is equl to 3280.84 (approximately). The circumference of the Earth is 3298.68 X 13 = 42882kms which is close (+7%) to modern value of 40075 kms.

Yuga theory (division of time):

Āryabhaṭa used the yuga theory to established the motions of heavenly bodies. The usual system of the yuga theory for division of time on a macrocosmic scale, according to the smṛtis and purāṇās as also the Sūryasiddhānta is as follows:

1 Kalpa	=	14	Manus
1 Manu	=	71	Mahāyugas
1 (Mahā) yuga	=	43,20,000	years

After pondering over the above calculation presented in our ancient granthas, Āryabhaṭa dispensed with this queer traditional theory of the yugas. He replaced it with a simple and astronomically viable theory as follows:

1 Kalpa	=	14	Manus

1 Manu	=	72	Mahāyugas
1 (Mahā) yuga	=	43,20,000	years

In this arrangement , 1 kalpa =1008 Mahāyugas instead of 1000. Since 1008 is divided by 7, every kalpa commences on the same weekday. Āryabhaṭa completely dispensed with time spent in "creation" and the "twilight" (sandhya) periods.

Further, Āryabhaṭa divided Mahāyuga into four parts but of equal durations unlike the traditional division of 4:3:2:1. For astronomical computations this equal division of 10,80,000 years each is more appropriate since in this period all planets complete integral numbers of revolutions. In other words, at the commencement of each yuga, the planets would all be in conjunction at the beginning of the zodiac. May be, Āryabhaṭa hit upon the scheme of equal division into 4 parts of a Mahāyuga since 4 happens to be a common factor for the numbers of revolutions of planets in a Mahāyuga. Thus Āryabhaṭa's yuga division reduced the size of huge integers involved for easily avoiding inconvenient vulgar fractions.

The nomenclature of Week-days:

Kālakriyāpāda of Āryabhaṭīya provides a rationale for naming week days as Bhanuvar, Somvar. Etc. after mentioning the relative position of planets:

> *Bhānāmadhaśśanaiścarasuraguru*
> *bhaumārkaśukrabudhacandrāḥ* ।
> *teṣāmadhaśca bhūmirmedhībhūtā khamadhyasthā*[15] ॥

Viz. The asterisms are the outermost. Beneath the asterisms lie the planets, Shani (Saturn), Guru (Jupitar), Mangala (Mars), Bhanu (the Sun), Shukra (Venus), Budha (Mercury) and Soma (the Moon) one below the other; and beneath them all Bhumi (the Earth) like the hitching peg in the midst of space.

Āryabhaṭa suggested that the world order commenced on

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[15] Kālakriyāpāda, 15

Shanivar, which was supported by Vateswara (904 AD). Āryabhaṭa's rationale for lords of the hours and days has been presented in the following shloka:

> *Saptaite horeśāśśanaiścarādyā yathākramaṁ śīghrāḥ* ।
> *Śīghrakramāccaturthā bhavanti sūryodayād dinapāḥ*[16] ॥

"The seven planets begging with Saturn which are arranged in the order of increasing velocity, are the lords of successive hours. The planets occurring for in the order of increasing velocity are the lords of successive days, which are reckoned from Sunrise". This calculation has been already elaborated in the topics **"Contribution of Āryabhaṭīya in alphabetical representation of numerals and numbers" and "The velocity of planets in a Yuga" (1 Yuga = 43,20,000).** These assertions on speed are also valid with the knowledge today in specific dimensions.

Thus, we see that the Āryabhaṭīya of Āryabhaṭa has contributed a lot in the landmark of modern universal mathematics and astronomy. It is a matter of glory that India's first satellite which was put into orbit on 19th April 1975, was named after great mathematician and astronomer Āryabhaṭa.

We may quote some thoughts of mathematicians and scientists. French mathematician Pierre Simon Laplace (1749-1827) had said "it is India that gave us the ingenious method of expressing all numbers by means of 10 symbols, each symbol receiving a value of position as well as an absolute value. The idea escaped the genius of Archimedes and Apollonius".

Albert Einstein has marked the Indian contribution *"We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made."*

Indian Scientist Dr.A.P.J. Abdul Kalam has written about the importance of ancient Sanskrit literature: "*Ancient Sanskrit literature is a store-house of Scientific principles and methodology. The work of our ancient scholars should be thoroughly examined and where possible integrated with modern science"* (Ignited Minds, p. 87).

BIBLIOGRAPHY

1. Acharya, Sudumna, 2006, *Ganita-Shastra ke Vikas ki Bharatiya Parampara* , Motilal Banarasi Dass, Delhi.
2. Archak, K.B. & Micheal (ed.),2007, *Science, History, Philosophy and Literature in Sanskrit Classics*, Sundeep Prakashan, Delhi.
3. Arya, vedveer, 2014, *Indian Contribution to Mathematics & Astronomy*, Aryabhata Publications, Hyderabad.
4. Bag, A.K., 1979, *Mathematics in Ancient and Medieval India*, Chaukhamba Orientalia, Varanasi.
5. Banerjee, M.; Goswami, B. (ed.),1994, *Science and Technology in Ancient India*, Sanskrit Pustak Bhandar, Calcutta.
6. Chaubey, B.B (ed.), 2009, *Rigveda volume I & II*, IGNCA, New Delhi.
7. Clark, Walter E. (trans. & ed.), The Āryabhaṭīya of Āryabhaṭa, University of Chicago Press, Chicago Illinois.
8. Dwivedi, Sudhakar, 1927, *Bijaganitam of Bhaskara II*, Banaras Sanskrit series
9. Jha, Lashanlal, 2008, *Lilavati*, Chaukhamba Vidyabhavan, Varanasi.
10. Kalam, APJ, Abdul, 2002, *Ignited Minds*, Penguin Books India, Panchsheel Park, New Delhi.
11. Mishra, K.K. (ed.), 2003, *Sanskrit Vānmaya mein Vijñana ka ithsasa*, NCERT, New Delhi.
12. Murthy, Venkatesh, 2000, Glimpses of Mathematics from the Sanskrit Works, Rastriya Sanskrit Vidyapeeeth, Tirupati.
13. Prakash, Satya, 1965, *Founders of Sciences in Ancient India*, The Research Institute of Ancient Scientific Studies; New Delhi.
14. Raja, M. L., 2011, Actual Date of Āryabhaṭṭa, Publisher Avinash, Chennai.
15. Rangacharya, M., (ed),1912, *Ganitasarasangraha*, Madras.
16. Rao, Balachandra, S., 1994, *Indian Mathematics and Astronomy*, Jnana Deepa Publications, Banglore.
17. Sen, S.N.; Bag, A.k., 1983, *Sulbsutras*, Indian National Science Academy, New Delhi.
18. Sharma, Punita (comm.), 2008, *Vedangajyotish*, Nag Publication Delhi.
19. Sharma, Ram Swarup (ed.), 1966, *Brahmasphuta Siddhant of*

Brahmagupta, Indian Institute of Astronomical and Sanskrit Research, New Delhi.

20. Sharma, Satyadev (comm.), 2008, *Aryabhatiyam*, Chaukhamba Surbharati Prakashan, Varanasi.
21. Shastri, Ramkrishna (trans.), 1999, *Yajurveda (shukl*a*yajurveda)*, Chawkhambha vidyabhawan,Varanasi.
22. Shukla, K.S., 1976, Āryabhaṭīya of Āryabhaṭa with comm. of Bhaskara I, Indian National Science Academy, New Delhi.
23. Shukla, K.S., 1976, Āryabhaṭīya of Āryabhaṭa with comm. of Bhaskara I, Indian National Science Academy, New Delhi.
24. Shukla, K.S., 1986, 'Vateswara Siddhanta and Gola of Vateswara, Indian National Science Academy, New Delhi.
25. Tirtha, Bharati Krishna, 2005, Vedic Mathematics, Motilal Banarasi Dass, Delhi.
26. Whitney, W.D. (trans.), 1996, *Atharvaveda*, Motilal Banarasi Dass, Delhi.