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THE *BAUDHĀYANA ŚULBASŪTRA*'S COMMENTARIES: EDITION, TRANSLATION AND EXPLANATIONS

1. Presentation of the Śulbasūtras, their commentaries and their editions.

The *Śulbasūtras* generally constitute one of the last chapters of the *Śrautasūtras*. Quoting Macdonell's *History of Sanskrit Literature* (p. 222-3), we can define them as "practical manuals giving the measurement necessary for the construction of the *vedi*, of the altars, and so forth. They show quite an advanced knowledge of geometry, and constitute the oldest Indian mathematical works". More recently, the *Dictionnaire des sciences*, edited in 1997 by Michel Serres and Nayla Farouki, describes (p. 410) as "practical geometry" the content of these "treatises on the construction of the sacrificial altars (...) in ancient India, after the 5th century BC".

The largest four of the extant *Śulbasūtras*, that is the *Baudhāyana*, *Āpastamba*, *Kātyāyana* and *Mānava Śulbasūtras*, have been edited by S. N. Sen and A. K. Bag in 1983, at the Indian National Science Academy, New Delhi. For the *Baudhāyana Śulbasūtra*, on which I will focus in this paper, these two scholars have used four manuscripts (from Benares, Munich, London and Ujjain) together with the first edition, published and translated into English by G. Thibaut in 1875 in the well known Benares periodical, *The Pandit*.

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1. At the time of the Conference, researcher at the Oriental Institute (University of Oxford) on a grant awarded by the Wiener-Anspach Foundation (1999-2000).

Some of the commentaries on the *Śulbasūtras* have already been edited (but not translated), for instance, those on the *Kātyāyana Śulbasūtra*, in Mysore in 1917, and on the *Āpastamba*, again in Mysore in 1931. There exist two commentaries on the *Baudhāyana Śulbasūtra*, one called simply *Śulbadīpikā* by *Dvārakānāthayajvan*, the other called *Śulbamīmāṃsā* by *Veṅkaṭeśvaradīkṣita*. Strangely enough, Sen and Bag (p. 12) attributed the *Śulbamīmāṃsā* to *Dvārakānātha*. Both commentaries were edited in Benares in 1979, by *Paṇḍit Vibhūtibhūṣaṇa Bhaṭṭācārya*, without translation, from manuscripts preserved in London, Benares and Poona, while the *Dvārakānātha*'s *Śulbadīpikā* was edited by Thibaut with his edition of the *Śulbasūtra*, but without translation. Thibaut used one manuscript from London and two from Benares, and has also given some excerpts of Veṅkaṭeśvara's *Śulbamīmāṃsā* from a grantha manuscript in London, which perhaps explain Sen and Bag's confusion.

Our aim, with Pr. Filliozat, whose help is invaluable and must be praised here, is to give a complete translation of both commentaries and a new edition based on all the manuscripts of which we have been able to obtain copies. For *Veṅkaṭeśvara*'s *Śulbamīmāṃsā*, we are using two manuscripts from London, one from Poona and one from Calcutta. There is one more manuscript in the Asiatic Society of Bengal, written in Bengali characters, of which I have not yet been able to obtain a copy, and if anyone reading this paper would be able to help me in that regard, I will be very grateful. For *Dvārakānātha*'s *Śulbadīpikā*, we use copies of 16 different manuscripts, some of them incomplete, coming from London, Madras, Poona, Wai, Bombay, Baroda, Ujjain, Chandigarh and Calcutta. But we are still missing copies of manuscripts from Alwar, Benares, Hyderabad and Lahore. As for the first commentary, help is needed, especially for Alwar, where I was not even allowed to see the manuscript.

2. The commentators

Veṅkaṭeśvara Dīkṣita (or *Makhin*) lived at the end of the 16th century in Tamilnadu, in a circle of well-read scholars, under the protection of the *nāyāka* of *Tañjāvūr*. His father, *Govinda Dīkṣita*, and his brother, *Yajñanārāyana Dīkṣita*, a famous poet, belonged to the same circle. Another famous poet, *Nīlakaṇṭha Dīkṣita*, was his pupil [2]. *Veṅkaṭeśvara* composed several commentaries, all connected with the *Mīmāṃsā*: a commentary on the *Ṭupṭikā* of *Kumārila*, on the *BaudhāyanaŚrautasūtra*, and a *Śulbamīmāṃsā*. As its title perhaps shows, this work is more concerned with issues in the ritual than with mathematical problems. Nevertheless, it also contains a mathematical commentary, unhappily often reduced to mere results and much smaller than *Dvārakānātha*'s *Śulbadīpikā*.

We know nothing of *Dvārakānātha*, except that he was the son of *Bhaṭṭa*, and younger than *Āryabhaṭa*, whose rules he quotes. Thus his earliest possible date is the 6th century. We have no terminus ad quem, except perhaps the date of *Veṅkaṭeśvara*, who often (but not always) gives the same results as *Dvārakānātha*. On the other hand, I must mention A.K. Bag again, who, in his *Mathematics in Ancient and Medieval India* (p. 139), places *Dvārakānātha* between the 5th and the 8th centuries, exactly in the same interval that he assigns to *Karavindasvāmin*, the earliest commentator on the *Āpastambaśulbasūtra*. Unhappily, Bag does not give the reason why he postulates this terminus ad quem.

I won't give any example here of the '*mīmāṃsical*' arguments *Veṅkaṭeśvara* develops, but, as a mathematician, I will rather deal with *Dvārakānātha*'s commentary, especially on the first chapter of the *Baudhāyanaśulbasūtra*, the chapter considered 'less practical'. But before that I will give some insight on the reasons why these mathematics were necessary to the Vedic ritual.

3. The śyena altar and its enlargement

For some very large Vedic sacrifices that are supposed to fulfill the *yajamāna*'s wish to attain heaven, defeat his enemies, get food or cattle, etc., the *Taittirīya Saṃhitā* (V.4.11.1-3) enumerates the shapes of the altar which is to be built in place of the *uttaravedi*, that is at the east end of the *mahāvedi* [3]. These sacrifices are called *kāmya*, in that

2. I owe all these biographical data to Pr. Filliozat's *Oeuvres poétiques de Nīlakaṇṭha Dīkṣita*, Pondichéry, 1967.

3. See the "Plan of sacrificial ground" given in the *Śatapatha Brāhmaṇa* (ed. J. Eggeling), part II, p. 475, *Sacred Books of the East*, vol. XXVI, 1885 (reprint by Motilal Banarsidass, 1978).

they are enjoined for specific desires (*kāmas*), and the construction of the altar, which is made up of 5 layers, each having 200 bricks, with all its auxiliary rituals, is called the *agnicayana*.

One of the shapes for this altar is the *śyena*, which is described in the *Baudhāyana Śulbasūtra* (III.8-9) as a flying bird of prey, generally translated falcon. It has in fact two variants, one with curved wings, and one with squares (see Figure 1). About its size, the *Śulbasūtra* (II.1-5) enjoins an area of 7,5 *puruṣas*^{2}, where the *puruṣa* is defined as the height of the *yajamāna* standing with his arms raised above his head. In the case of the square *śyena*, there would be 4 *puruṣas*^{2} for the *ātman*, 1 for each wing and the tail, 0.2 for the lengthening of each wing, and 0.1 for the lenthening of the tail.

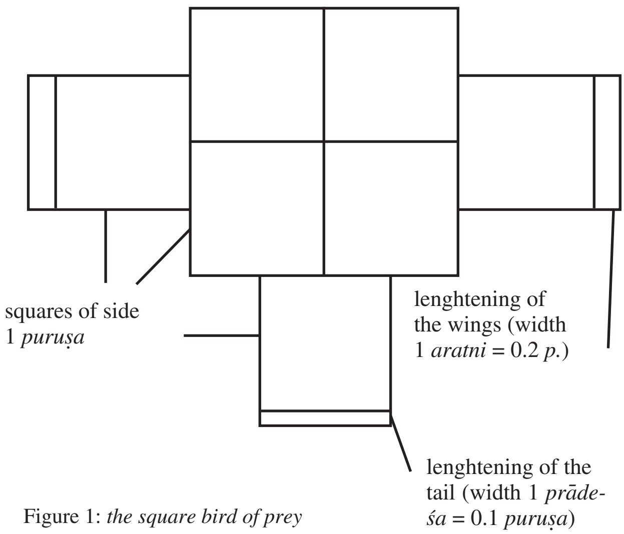


Figure 1: *the square bird of prey*

It is also enjoined that all the other altars should have the same area, even if they have a different shape. This poses the interesting mathematical problem of transforming one figure into another without altering its area (for instance, think of the famous "squaring the circle" problem). But there is more. It is also enjoined that, if the same *yajamāna* wishes to offer the same sacrifice again, then the altar should have an area of one more *puruṣa*^{2}, and so on. This is another mathematically interesting problem: how to transform a given figure into a homothetical one, of which area is increased by a fixed quantity.

We will see several examples of the first problem, but let us now look at how the *Śulbasūtras* solved the second one. The *Baudhāyana Śulbasūtra* (II.11-13) proposes to cut the additional *puruṣa*[2] into 15 identical rectangles and then to add two of them to one of the initial squares (see Figure 2). This is straightforward, for remember that the square *śyena* is always divided into 7 1/2 parts of the same area. So, when two such rectangles are added to one of the seven initial squares, it yields a rectangle, which one must tranform into a square in order to build the shape of the following altar. This requires a rectangle-square transforming rule, which does not alter the area. In fact, this is already an example of the first problem mentioned above. It is given in the first chapter of the *Baudhāyana Śulbasūtra*, which we will now consider more accurately.
To begin, let us first have a look at the organization of the first chapter.

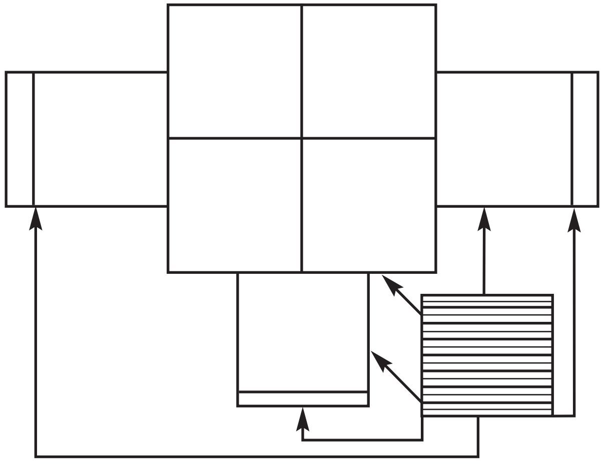
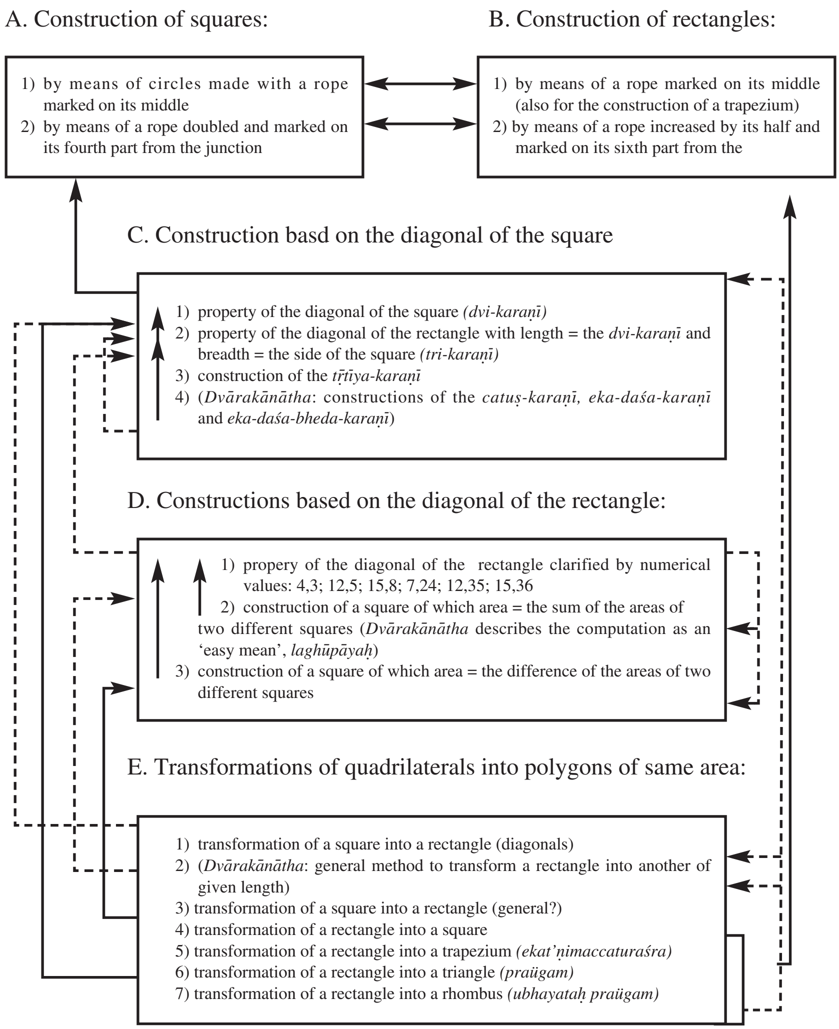


Figure 2: *increasing the square bird of prey*

4. The first chapter of the Baudhāyanaśulbasūtra

The outline given below does not show the beginning of the chapter, which is concerned with the definition of the different units, such as the *puruṣa*, the *aṅgula* 'finger's breadth', equal to 1/120 of the *puruṣa*, the *aratni* 'cubit' [4], equal to 24 *aṅgulas* (it is the lengthening of the wings), the *prādeṣa* ‘span’, equal to 12 *aṅgulas* (it is the lengthening of the tail), the *pada* ‘foot’, or the tila ‘sesamum indicum’ or sesam seed, which is strangely defined as the 34th part of an *aṅgula*.



4. French ‘coudée’, much used about ancient Egyptian measurement. The *Harrap’s Shorter /Bordas French/English/French Dictionary* (reprint 1975) has both entries.

F. Circulature and quadratures:

1) construction of a disc by means of the side and diagonal of a square (circulature of the square, described as *anitya* by *Dvārakānātha*)

2) first construction of a square of which side is the 1 - 28 / 8 x 29 - (1 / 6 x 8 x 29 - 1 / 8 x 6 x 8 x 29) [= 7 / 8 + 1 / 8 x 29 - (1 / 6 x 8 x 29 - 1 / 8 x 6 x 8 x 29)]th of the diameter of a disc (quadrature of the circle)

(*Dvārakānātha* replaces the fraction 7 / 8 + 1 / 8(41 / 1392) or 1 - 1 / 8 (1351 / 1392); *Veṅkaṭeśvara* gives an incorrect fraction)

3) second quadrature of the circle by means of the 13 / 15th of the diameter of the cicle (*Baudhāyana* describes it as *(a)nityā; Dvārakānātha* says *sthūla* 'gross', and proves the inaccuracy of F.1), the non-reciprocity of F.1) and F.3), and proposes improvements; *Veṅkaṭeśvara* glosses *anityā* by *ananugatā* 'not tallying')

4) value given to the quotient of the diagonal of the square to its side: 1 + 1 / 3 + 1 / 12 - 1 / 12 x 34 (numerical value described as *(pari)jñānopāya* and clarified by means of the square of side 12 aṅgulas by both commentators)

value 7 / 5 given to the quotient (*Mānava Śulbasūtra* and *Baudhāyana* indirectly)

The outline focuses on the mathematical features of the chapter, without refering to the objects for which they are meant to be used. These objects are described at the end of the first chapter, which is devoted to the dimensions and positions of the different agnis and *vedis* and to the numerous objects that have to be built on the *mahāvedi*: *dhiṣṇyas*, *havirdhāna*, *uparavas*, *uttaravedi*, etc. The two following chapters deal with the construction of the different types of *kāmya* altars.

We thus begin with E.4, which explains how to transform a rectangle into a square.

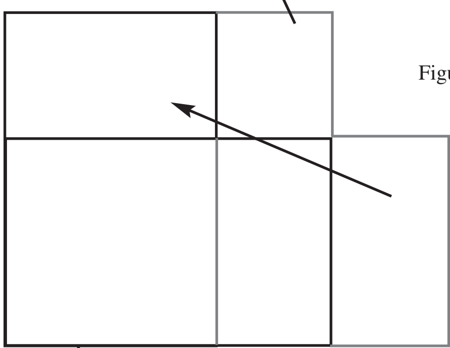


Figure 3: *quadrature of the rectangle*

"If one wishes to square a rectangle, after having made of the breadth the side of a square, one should place the rest, divided into two equal parts, on the two sides of the square. One should fill the void by inserting a square. The deduction of this /square/ has been explained." (Baudh. Śulb., I.54)

So, the initial rectangle has been transformed into what the historians of Greek mathematics use to call a gnomon, that is, the difference of two squares. To complete the transformation into a square, *Baudhāyana* refers explicitely (by the words *tasya nirhāra uktaḥ* 'the deduction of this /square/ has been explained') to another construction, that is D.3: the construction of a square of which the area equals the difference of the areas of two different squares.

There is a certain order in the propositions shown on the outline. The relations between them are in certain cases emphasized by the *Śulbasūtra* itself, as we just saw between E.4 (I.54) and D.3 (I.51) (plain arrows), or by the commentators (doted arrows).

Now, *sūtra* I.51 (D.3) reads:

"If one wishes to subtract a square from a square, one must cut from the bigger a strip having as breadth the side of the square one wishes to subtract; then one should bring diagonally into contact the length of the strip with the other side of it and, where it falls, should take away the rest of the side." (Baudh. Śulb., I.51)

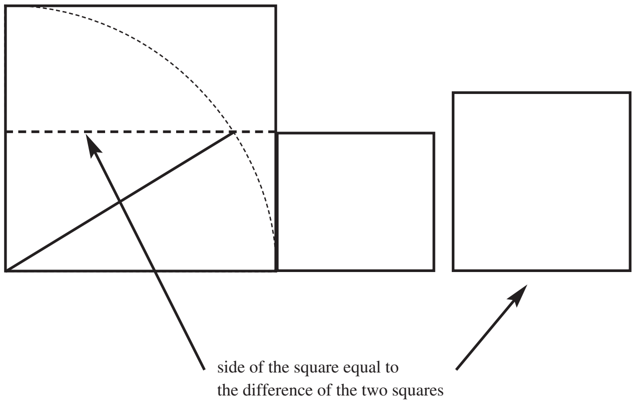


Figure 4: *difference of two squares*

Of course, there is also a *sūtra* (I.50 or D.2) to explain how to transform the sum of two squares into a square:

> "If one wishes to assemble two different squares, one must delineate with the side of the smallest /square/ a strip of the largest one. The diagonal cord of /this/ strip is the side /of the square made of the two squares/ assembled" (*Baudh. Śulb.*, I.50)

It has the following geometric interpretation:

These last two constructions are of course based on the so-called

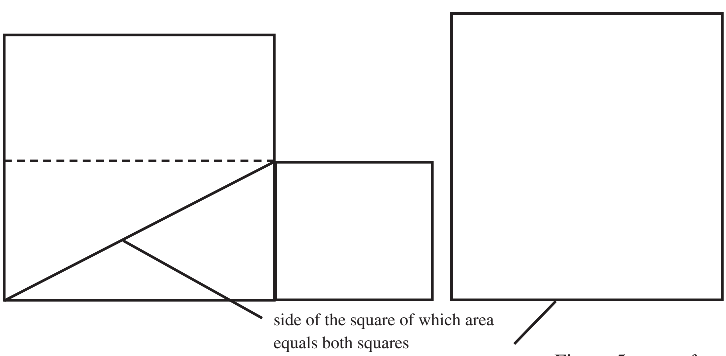


Figure 5: *sum of two squares*

Pythagoras theorem, stated in two parts: C.1 (property of the diagonal of the square) and D.1 (property of the diagonal of the rectangle), but it is the last proposition which is meant here, because the two squares have different sides. Here, *Baudhāyana* is not explicitly refering to D.1, but the fact that D.2 and D.3 (constructions of a square of which area is equal to the sum/difference of two squares) follow immediatly, in the order of the treatise, the property of the diagonal of the rectangle proves that they were considered as resulting directly from this property.

Let us now return to our altar increasing problem. The new basic square is easily built with the help of the rectangle-square transformation rule, but what if one doesn't begin with a 7.5 *puruṣas*^{2} altar but with a 8.5 *puruṣas*^{2} altar, or one still larger? In that case, one could transform the rectangle of area 2/15 *puruṣas*^{2}, which is to be added, into a square by E.4-D.3, and then add this little square to one of the large squares of the altar by D.2. But there still remains one problem: how should we increase the lengthenings of the wings and tail? We could, for instance (see Figure 6), transform the rectangle to be added and the initial lengthening into squares by E.4-D.3, then add these two squares by D.2, but what then? We are left with a square that has to be adapted to one of the seven large squares we have just constructed. At that stage, a square-rectangle tranformation rule would be most useful, for what we need to do in fact is to transform our square into a rectangle of a given length, without altering the area. A square-rectangle tranformation rule would also simplify our procedure, for we will then be able to adapt the little rectangle directly to the initial lengthening, in order to obtain the single rectangle, which now we simply have to adapt to the new square.

I do not say that *Baudhāyana* himself proceeded that way, and in

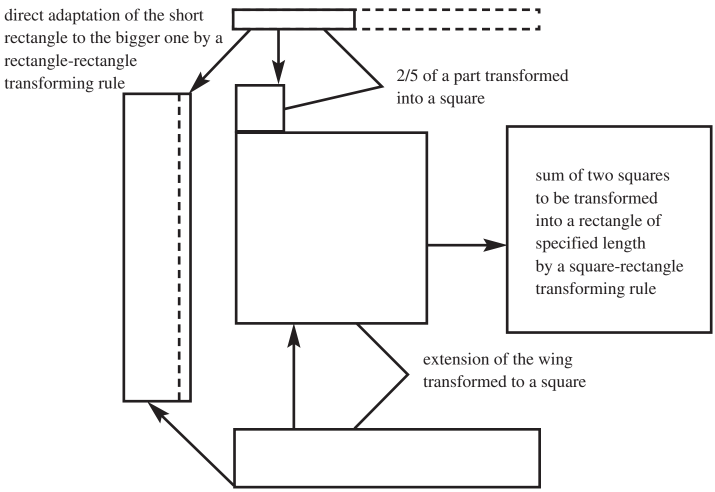


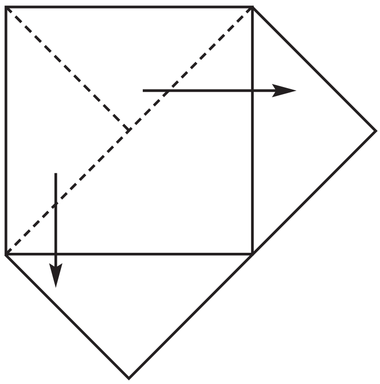
Figure 6: *sum of two squares*

fact it is likely that in practice, he proceeded like *Kātyāyana* (V.5), who, after having increased one large square, defines a new *puruṣa* as the side of that square.

However, *Baudhāyana* gives two different square-transformation rules, E.1 and E.3:

> "If one wishes to transform a square into a rectangle, after having cut it by the diagonal and divided in two fourths a part, one should place them properly on the two sides of the squares. (*Baudh. Śulb.*, I.52)
>
> In that case also, after having cut the square with this same rectangle, let one place the rest elsewhere." (*Baudh. Śulb.*, I.53)

The first one is very peculiar and is probably related to a very



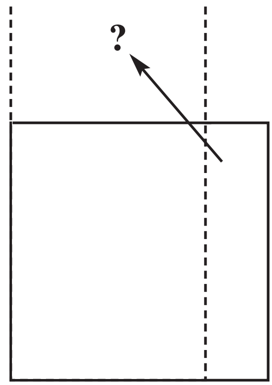
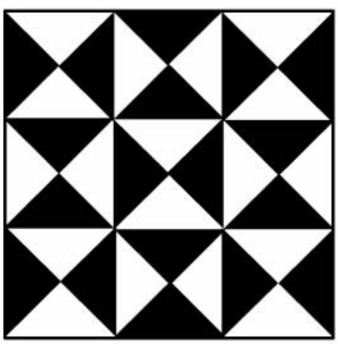


Figure 7: *transforming a square into a rectangle*

Figure 8: *Indus tiling*



ancient tiling (see Figure 7) that could have led to the discovery of the property of the diagonal of the square. This tiling appears already on some sherds of Mohenjo-Daro [5]. The second *sūtra* seems to be more general, for it considers the breadth of the rectangle to be built as arbitrary. But it is also disappointing, for it doesn't give a precise procedure to follow.

About this last *sūtra*, Georg Thibaut, the first editor of the *Baudhāyana Śulbasūtra*, wrote: "I do not venture on a translation of this *sūtra*, as it does not clearly appear to what geometrical operation it refers. Moreover there seems to be a lacuna in the (*Dvārakānātha*'s) commentary (...)" [6], but he finally [7] ventured to give an interpretation, which seems to correspond to what *Veṅkaṭeśvara*'s commentary of I.53 suggests [8]. *Dvārakānātha* himself

5. J. Marshall, *Mohenjodaro and the Indus Civilization*, 3 vols, Delhi, 1973 (reprint), pl. XCII, 10; 14-17.

6. *The Pandit*, June 1, 1875, p. 19.

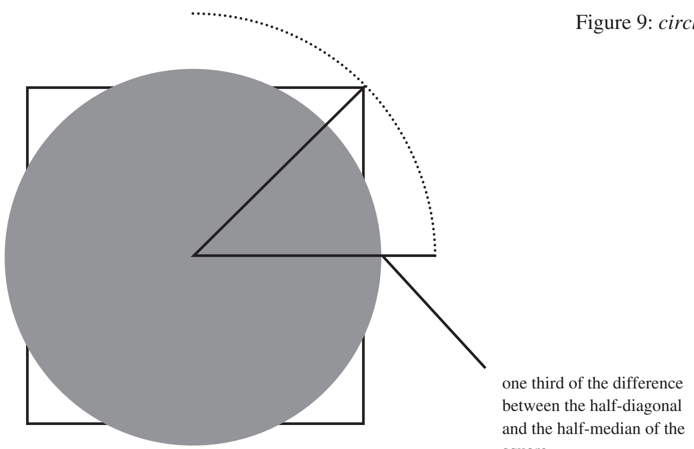
7. *On the Śulvasūtras, Journal of the Asiatic Society of Bengal*, vol. 44 (1875), p. 246.

gives the exact construction of the rectangle, not after the *sūtras* I.52-3, but rather after the *sūtra* I.51.

5. The approximations to the diagonal of the square

Now that we have considered these rectangle-square and square-rectangle transformation rules from the point of view of the increasing of the *kāmya* altars, we will focus on another set of problems, the search for approximations to √2 (more exactly, the quotient of the diagonal of the square to its side). A very accurate estimation (it is equal to √2 up to the fifth decimal or, more precisely, the relative error is ≈ 1.5×10^{-6}) is given at the end of the outline as being equal to the fraction 1 +1/3 +1/12 - 1/12x34. Let us first point out that this value was useful mainly for build-ing a quadrature procedure (described in F.2) from the "circle squaring" procedure (F.1, see Figure 9) given beforehand in the treatise, as Thibaut has already shown in his paper.[9] Both procedures are examples of the first type of problem.

Figure 9: *circling the square*



8. For more details, see J.M. Delire, *Quelques aspects arithmétiques des commentaires de Veṅkaṭeśvara et de Dvārakānātha à la géométrie du Baudhāyana Śulbasūtra*, to appear in *Oriens-Occidens*, Cahiers du Centre d'Histoire des Sciences et des Philosophies Arabes et Médiévales, vol. 4.

9. Op. cit., pp. 253-4.

The computations for deriving F.2 from F.1 with the help of F.4 are rather involved and I will take here another example in order to explain the procedure. After the quadrature F.2, *Baudhāyana* gives another one, F.3., much grosser than F.2. This quadrature seems to be more ancient and is quoted by him simply because of its 'traditionality' (*nitya-tva*). It equates the side of the square to be built with 1 - 2/15th of the diameter of the circle, and could have been derived, and here I am reconstructing, from another, grosser, approximation of ṅ2, given in the *Mānava Śulbasūtra* (10.1.2), that is 7/5. Suppose we apply the circulature procedure F.1 to a square of side 1 and diagonal supposed to be 7/5: the part of the diagonal exceeding the square would equal 2/5. Taking the third of this and adding it to 1 would yield 1 + 2/15 for the quotient of the diameter of the disc to the side of the square, a numerical equivalent to the circulature F.1. Now, to build a numerical quadrature, the Indians had only to find the inverse of the fraction 1 + 2/15, which, at a time when they were probably not yet as expert in fractions as *Baudhāyana* was, could have been considered to be 1 - 2/15 (in any case, the product (1+2/15)(1-2/15) misses 1 by less than 0.02 (2%), which is not so bad).

Let us now return to our accurate approximation of √2 and try to understand how this accurate fraction could have been obtained. We will have the opportunity again to go through some of the most interesting mathematical procedures of *Baudhāyana*, to observe how he connected them with each other, and how the commentators understood and, in some cases, improved these procedures.

The problem is the following one: finding √2 is, geometrically speaking, the same as transforming a rectangle of area 2 into a square. We thus begin with E.4, which we have already met. The initial rectangle is thus transformed into a gnomon, which is the difference between a square of side 1 ½ and a square of side ½ (see Figure 10). To complete the transformation into a square, we have already seen that *Baudhāyana* refers explicitely to another construction, that is D.3: the construction of a square of which area equals the difference of the areas of two different squares.

These two constructions are basically geometrical, and even the property of the diagonal of the rectangle D.1, on which they are based, is described as a geometrical construction: 'what the length and breadth produce (*kurutaḥ*) separately, both of those the diagonal cord of the rectangle produces (*karoti*) (*dīrgha-caturasrasya akṣṇayā rajjus pārśvamānī tiryaṇmānī ca yat pṛthag-bhūte kurutas tad ubhayam karoti*).

But we could also apply these procedures to numbers. *Baudhāyana* himself gives the numerical examples 4,3; 12,5, etc., to illustrate this property, and *Dvārakānātha* comments upon these values. He refers first to C.2, I think in order to complete the pairs 4,3 by 5, 12,5 by 13, etc., because in C.2. are given the two sides of a rectangle (*pramāṇa*, i.e.1 and *dvi-karaṇī*, i.e. √2) together with its diagonal (*tri-karaṇī*, i.e. √3).

But *Dvārakānātha* goes further, saying: "there is a diagonal cord of five when the sides are three and four (...); when two of them are known, the third could be known; so when the breadth (*pārśvamānī*) and length (*tiryaṇmānī*) are known, by squaring them separately (*pṛthag-vargayitvā*) and adding (*saṃyojya*) them, the square root (*varga-mūla*) is the diagonal cord; when the breadth and diagonal cord are known, by subtracting (*viśodhya*) the square (*varga*) of the breadth from the square of the diagonal cord, the length is the square root of the rest ((*vi*)*śiṣṭa*), ..."

We observe here that the terms themselves refer to numbers: *varga* and not *caturaśra*, *varga-mūla* and not *karaṇī*, which remind us of *Āryabhaṭa*, whose *Gaṇitapāda* was well known by *Dvārakānātha*.

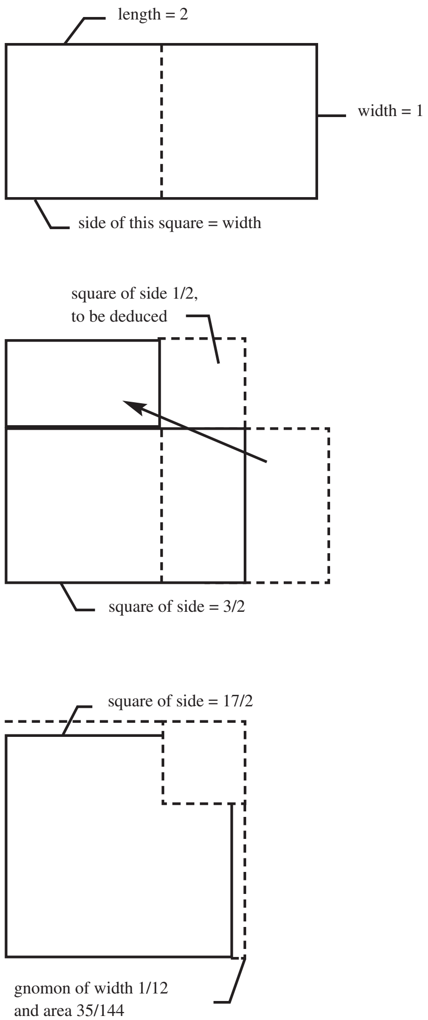
Let us now try to describe the procedures E.4 and D.3 numerically: a first number is obtained when one applies the beginning of E.4, that is, cuts into the rectangle of length 2 and breadth 1 a square of side 1. 1 (one) is the first approximation to √2.

Then, applying the rest of E.4 would yield a gnomon of larger side 3/2 (three halves), which is the second approximation.

Of course, what we really need is a square and not a gnomon, but we know that the geometrical procedure D.3 is useless when working with numbers. However, we can go further by generalizing our two first steps. Indeed, we could describe the side 3/2 (the second approximation) as being obtained by adding to the first approximation 1 the remaining area (when the square on/of the first approximation has been removed) divided by twice this first approximation, because what we need now to add is half the length of this remaining area (see Figure 10).

In order to obtain another numerical value, we could generalize

‘Geometrical’ transformations (*Śulbasūtra*)



Arithmetical formulas (*Bakhshālī* manuscript)

$$K = p_1{}^2 + E \qquad (1)$$

$$(K = 2_1\ 9_1 = 1_1\ E = 1)$$

$$p_2 = p_1 + \frac{E}{2p1} \qquad (2)$$

$$\left(\frac{E}{2p1} = \frac{1}{2},\ p_2 = \frac{3}{2}\right)$$

$$p_3 = p_2 - \left(\frac{E}{2p1}\right)^2 / 2p_2 \qquad (3)$$

$$\left(\left(\frac{E}{2p1}\right)^2 / 2p_2 = \frac{1}{12},\ p_3 = \frac{17}{12}\right)$$

Figure 10: *the procedure applied to numbers*

what we have just done. There is a remaining area (or rather an area in excess, in that case), that is $^1/_4$, the area of the little square, which has to be removed from the big square. If we divide this area by twice the previous approximation, that is 3, and subtract the result from this previous approximation, we obtain 3/2 - (1/4)/3 = 3/2 - 1/12 = 17/12.

We can thus define, in our quest to improve an approximation, the fundamental step as being: *to subtract from the initial area of the rectangle the square on/of the approximation, to divide the difference by twice the approximation, and then to add (which is actually to subtract when the difference is negative) the result to the approximation.* [10]

A further such step would lead to the approximate value given by *Baudhāyana*, because the square of side 17/12 has an area in excess of 1/144 over the initial rectangle. Dividing this excess by 2x17/12 would yield 1/34x12, to be subtracted from 17/12.

6. Extracting the square root

The numerical procedures leading to the three first approximations could be generalized to the extraction of the square root of any number. These three procedures occur also in the *Bhakshālī* manuscript. On the right (Figure 10) are the three 'formulas' by which Takao Hayashi, the editor of the *Bhakshālī* manuscript, represent these procedures. Hayashi himself suggested [11] that the *Bhakshālī*'s formulas could have been derived from the *Śulbasūtra*'s method of tranforming a rectangle into a square.

Of course, 17, as an approximation to the diagonal of a square of side 12, could have been simply 'guessed' by observing that $2x12^2 = 17^2-1 \approx 17^2$. [12] The commentators very often take a square of side 12 *aṅgulas* to exemplify *Baudhāyana*'s procedures (both in A.2, *Veṅk.* in

10. The fundamental step could also have arisen from the following question, generalizing E.4: what if we have to add 'numerically' a small square (or a rectangle) to a larger one? We could, following E.4, cut it into two equal parts and, then, adapt both parts to two adjacent sides of the larger square. This is easily done by dividing their area (equal to half the area of the small square or rectangle) by the side of the larger square.

11. *The Bakhshali Manuscript*, Groningen, 1994, p. 107.

12. An alternate method of producing the fraction (F.4) has been given by Datta, *The Science of the Śulba, a study in early Hindu geometry*, Calcutta, 1932, pp. 192-4. Instead of following *Baudh. Śulb.* I.54 by cutting the second (remaining) square in two parts, he cuts it in three equal parts in order to get the 1/3, and the 1/12 afterwards, of the fraction. But to explain the 1/12x34, he has recourse to the transformation of the little additional square into two strips, as we do.

B.1, both in B.2, etc.; in C.2, *Dvār*. uses a rectangle of breadth 12 *aṅg*., length 17 *aṅgulas* - 1 *tila* and computes the diagonal as 20 *aṅgulas* + 17 *tilas*), but even if *Baudhāyana* or another *śulbaka* derived 17 *aṅgulas* - 1 *tila* as the approximate diagonal of a square of side 12, it remains to explain what caused him to define the *tila* as being the 34th part of an *aṅgula*. The use of the *tila* would have made the computation (for instance, the verification that $(17x34 - 1)^2 = 2x(12x34)^2 + 1$) free of fractions. But finally, in order to make the approximation con-venient to any square (for which the commentators use the word *abhilaṣita*, as *Veṅk*. in A.1, B.1 and the beginning of B.2, or *abhīṣṭa*, as *Dvār*. in B.1), it was necessary to divide 17 *aṅgulas* - 1 *tila* by 12 *aṅgulas*, which yields 17/12 (transformed into the sum of unitary fractions (12 + 4 + 1)/12) - 1/(12x34), therefrom the quotient as expressed in F.4.

Let us now look at the general extraction procedure, to which Āryabhaṭa devotes a rule:

bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena /
vargādvarge śuddhe labdhaṃ sthānāntare mūlam //

Hayashi's translation is "[Having subtracted the greatest possible square from the last square place (i.e., odd place),] one should always divide the non-square [place] by twice the square root. While the square [of the quotient] is subtracted from the square [place], the quotient is [put down] at the next place as [part of] the square root."

But Rodet gave [13] a slightly different translation:

"One shall always divide the 'non square section' (avarga) by twice the root of the 'square section' (varga) [which precedes], after having subtracted from this 'square section' the square of the root; the quotient is the root at a one place's distance."

Ārybhaṭa's rule is essentially the fundamental step adapted to the decimal positional numeration. Without any doubt, *Dvārakānātha*

13. Léon Rodet, *Leçons de calcul d'Āryabhaṭa, Journal Asiatique*, mai-juin 1879, pp. 393-434: "On divisera toujours la «tranche non carrée» par le double de la racine de la «carrée» [qui précède], après avoir retranché de cette «carrée» le carré de la racine; le quotient est la racine … distance d'une place". »

used it whenever he had to compute a square root, for the following reasons. Firstly, because he knew *Āryabhaṭa*'s work, from which he is explicitely quoting two rules, and, secondly, because he insists on using the numerical procedure of extraction (see the above quoted text), even calling it once *laghūpāya* when he explains numerically the procedure of adding the areas of two squares (D.2):

"Here, an easy method: by squaring separately the side of the big square and the side of the little square, adding them, and taking the square root (mūlamānīya) *of it, the two square areas are put together in a square constructed with a cord of this length."*

Moreover, in another part of his commentary (concerning the construction of the *paitṛkī vedi*), when computing the side of a square of the same area as the *mahāvedi*, that is 972 *padas*^{2}, he is explaining that one should first multiply by 225 (that is 15^2, in order to convert the *padas*^{2} to *aṅgulas*^{2}), then by 1156 (that is 34^2, in order to convert the *aṅgulas*^{2} to *tilas*^{2}), before taking the square root. By this, he adapts the very peculiar numerical system of the *Śulbasūtras* to the decimal one. He calls this method *jñāna-prakāra* ('way to know'), which reminds of the name (*pari*)-*jñānopāya* given by both commentators to the numerical approximation (F.4) of $\sqrt{2}$.

7. An example from Dvārakānātha

For the extraction of the square root of 972 *padas* = 252817200 (*tilas*^{2}), *Dvārakānātha* probably began with 15. The correspondant *avarga* is 2528, from which 225. must be subtracted: result 278, to be divided by 30, which yields 9.

The following approximation is thus 159, with square 25281, and the corresponding *avarga* is 252817. One subtracts 25281.: result 7, to be divided by 310, which yields 0.

The next approximation is 1590, with square 2528100, and the corresponding *avarga* is 25281720. On subtracts 2528100.: result 720, to be divided by 3180, which yields 0.

The square root is thus 15900 = 31//2//22, which is not equal to

Dvārakānātha's result (31//2//26). Two mistakes could have occurred at the last step: instead of subtracting 2528100. from 25281720, he could have subtracted 2528100. from 252817200: result 7200, which, instead of dividing by 3180, he could have divided by 1590 (the last approximation and not its double), obtaining 4.

It is difficult to believe that *Dvārakānātha* could have made such mistakes, but, while the haste to finish the computation could explain the second, the first one could have occurred from the very automatic character of the procedure, as described by *Āryabhaṭa*. At each step, one subtracts the square of the approximation from the corresponding *avarga* without taking into account their respective size, simply by placing them one below the other, as we do in a division.

All the computations could be done in the following way:

v/av/av/av/av
2/52/81/72/00 (*varga* 225 of) square root 15
(-2/25/.
278 to be divided by 30, result 9; *varga* 25281of) square root 159
(2/52/81/72/00
-2/52/81/.
7 to be divided by 318, result 0; *varga* 2528100 of) square root 1590
(2/52/81/72/00
-2/52/81/00/.
72/00 (mistake: 7200 instead of 720)
divided by 1590 (mistake: 1590 instead of 3180),
result 4, instead of 0) final root 15904 (instead of 15900).

Of course, in order not to repeat 2/52/81/72/00 at each step, one erases the intermediary computations (in brackets) to keep only the last approximation. [14]

14. Another way, which avoids squaring the entire approximation at each step and replaces it by two easier steps, is used by the commentators, from *Bhāskara* I (c.628 AD) on. An example of this disposition can be found in Datta & Singh's *History of Hindu Mathematics*, 1935, vol. I, pp. 173-5.

Conclusion

After having pointed to the connections made by *Baudhāyana* himself, or by the commentators, between the different procedures described in the *Śulbasūtra*, that is to its structure, and given some extracts of the commentaries in order to clarify these procedures, I will only emphasize some points. There is in the *Baudhāyana Śulbasūtra* a real concern for inverting the procedures, even though the inverted ones are in some way useless. For instance, we have seen that the approximate values of √2 (F.4 or *Mānava*'s 7/5) were meant to devise a quadrature from the circulature known to all the *Śulbasūtras*, but can one imagine that the huge fraction (F.2) so obtained by *Baudhāyana* would have been of any practical use? We may also mention again the transformation of a square into a rectangle (E.1 and 3), inverse to the procedure (E.4). It seems that this transformation was not practically necessary, but only a manifestation of *Baudhāyana*'s concern to complete his mathematical corpus. We could also describe the *Baudhāyana*'s quadrature as an example of the search for mathematical precision beyond the practical 'discernability'.

After having insisted on these features of the *Baudhāyana Śulbasūtra*, which are only a small part of what could be said about the mathematical achievements of the *Śulbasūtras* in general, my final conclusion would be that these works certainly deserve more than the somewhat contemptuous designation of 'practical geometry' they have been credited by Michel Serres, himself a great admirer of Greek geometry, in his *Dictionnaire des Sciences*.

15. For this we find other examples, such as the four different proce-dures invented to place the *dakṣiṇāgni* at its proper position south-east of the *gārhapatya* and at a distance from the *āhavanīya* twice its distance from the *gārhapatya*. None of these procedures yields the right place, but the most accurate has a relative error of 0.72% in distance (between the *dakṣiṇāgni* and the *gārhapatya*) and an angular error of 0°22'54" (to the west).